LOCALIZATION OF HEAT GENERATION IN A DIELECTRIC UNDER THE ACTION OF AN SHF ELECTROMAGNETIC FIELD

R. G. Ruginets and R. Sh. Kil'keev

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We examine the thermal processes within a dielectric, brought about by the action of an electromagnetic SHF field on the material.

The intensity with which a dielectric is heated in an SHF field is determined by the dielectric losses and the power of the electromagnetic emission [1]. The dielectric losses, in turn, increase as the temperature rises, thus enhancing more intensive generation of heat. Depending on the properties of the material and the conditions of heat exchange, any further development of thermal processes within the dielectric may lead to accelerated heating of the material, with formation of local superheated zones in which the material melts down or is destroyed.

The melting is accompanied by a jumpwise increase in electrical conductivity and a loss of dielectric properties. As was demonstrated in [2], the existence of local superheated zones may be stable over a prolonged period of time, with the SHF radiation completely screened out by the melt.

The unique features involved in the thermal processes, associated with the formation of local heating zones, have been investigated by methods of numerical modeling. Experimental verification of the results from the modeling was performed on quartz ceramic specimens consisting primarily of silicon oxide ($SiO_2 \geq 98\%$). Ceramic specimens of this kind were placed into a rectangular waveguide of cross section 104 × 220 mm. Thin thermocouples were used to measure the temperature within the specimen. To eliminate SHF induction, the generator was switched off during the course of the measurements.

The data on the electrical and thermal properties of the ceramics, needed for the calculations, have been taken from [3-5], and they were used to construct the temperature relationships shown in Fig. 1.

To construct the mathematical model we examined a ceramic-material plate of thickness $\ell$ positioned in a short-circuited waveguide section of length L in which a standing electromagnetic wave is generated. For purposes of simplifying the problem, real waveguide modes were replaced by a plane monochromatic wave [6]

$$\frac{d^2\dot{E}(x)}{dx^2}+\omega^2\mu_0\varepsilon_0\dot{\mu}\dot{\varepsilon}(T)\dot{E}(x)=0 \tag{1}$$

with boundary conditions [1]

$$\dot{E}(0)-\frac{1}{j\omega\sqrt{\varepsilon_0/\mu_0}}\frac{d\dot{E}(0)}{dx}=2E_0,\quad \dot{E}(L)=0,$$

where $E_0 = \sqrt{2S_0\sqrt{\mu_0/\varepsilon_0}}$ is the amplitude of the electrical component of the incident wave.

The thermal field in the material is described by the nonsteady equation of heat conduction [7]

$$c(T)\rho(T)\frac{\partial T(x,t)}{\partial t}-\frac{\partial}{\partial x}\left[k(T)\frac{\partial T(x,t)}{\partial x}\right]=0{,}5\omega\varepsilon_0\varepsilon''(T)|\dot{E}(x)|^2 \tag{2}$$

with the boundary conditions

$$k\frac{\partial T(0,t)}{\partial x}=-a[T_0-T(0,t)]-zc_s[T_0^4-T^4(0,t)],$$

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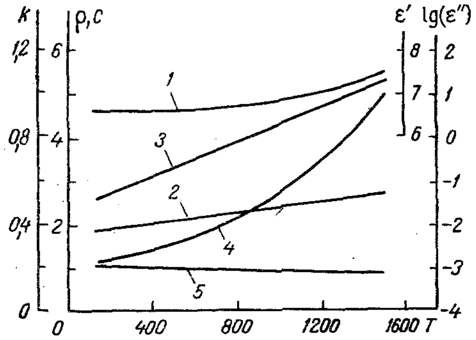


Fig. 1

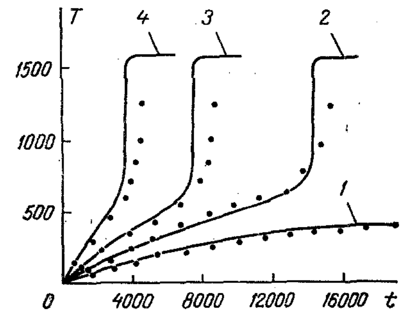


Fig. 2

Fig. 1. Thermal and electrophysical properties of quartz ceramics as functions of temperature (on the basis of data from [3-5]): 1) ε'; 4) ε''; 3) k, W/(m·K); 2) ρ, $10^3$, $kg/m^3$; 5) c, kJ/(kg·K). T, °C.

Fig. 2. Change in temperature for various densities of the SHF flow of power $S_0$ (the dots identify experimental data; $\ell = 0.2$ m, $\lambda_0 = 0.32$ m): 1) 50 kW/$m^2$; 2) 150; 3) 300; 4) 600.

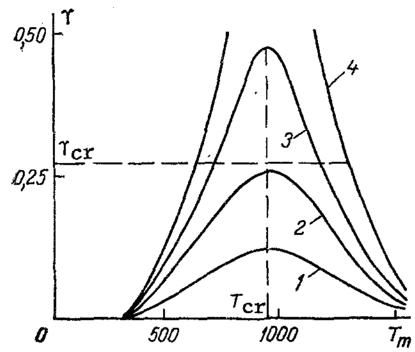


Fig. 3

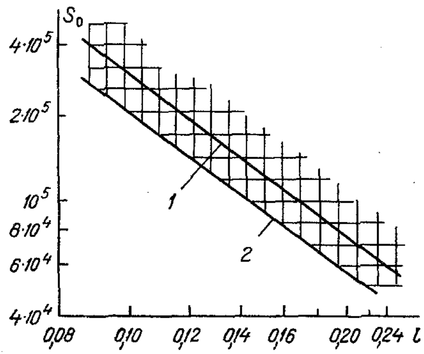


Fig. 4

Fig. 3. Criterion of volumetric heating as a function of the maximum steady-state temperature $T_m$, K: 1) A = $10^{-5}$ m/W; 2) $2\cdot10^{-5}$; 3) $4\cdot10^{-5}$; 4) $8\cdot10^{-5}$.

Fig. 4. Critical density of SHF power flux as a function of plate thickness under various conditions of heat exchange with the ambient medium: 1) forced convection, $a$ = 16.9, z = 0.93, $\gamma_{cr}$ = 0.214; 2) free convection, $a$ = 5.3, z = 0.93, $\gamma_{cr}$ = 0.273. $S_0$, W/$m^2$; $\ell$, m.

$$k\frac{\partial T(l,\ t)}{\partial x}=a[T_0-T(l,\ t)]+zc_s[T_0^4-T^4(l,\ t)].$$

The initial conditions $T(x, 0) = T_0$.

We used a numerical-analytical joining method to solve Eq. (1), and a finite-difference method to solve Eq. (2) [8].

Results and Discussion. The characteristic curves showing the changes in the maximum temperature in the central portion of the specimen for various levels of SHF power are shown in Fig. 2.

For small values of the SHF power flux density through the dielectric, the generation of heat within the specimen is not great. The temperature, increasing monotonically, tends to some limit value at which a steady-state thermal field is established (Fig. 2, curve 1). The temperature variations in the thermal and electrical properties are insignificant and exert no significant influence on the thermal processes.

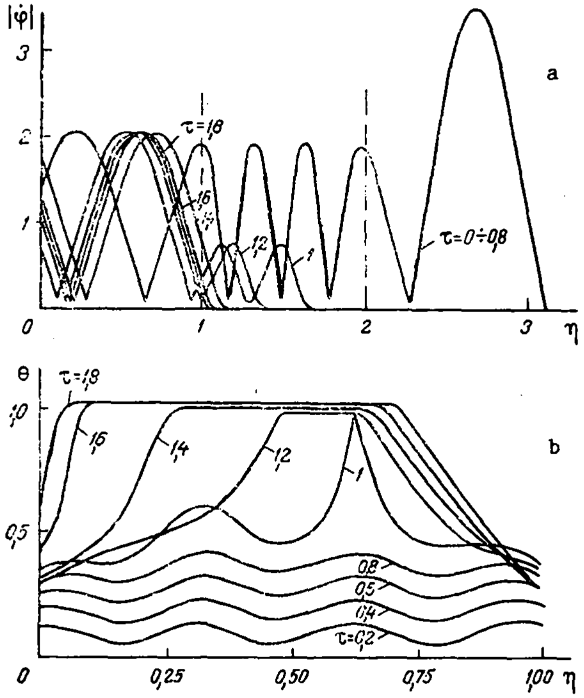


Fig. 5. Change in the electric and thermal field in the case of SHF heating under conditions of free convection ($S_0$ = 600 kW/m², L = 0.6 m, ℓ = 0.2 m, $t_0$ = 3210 sec): a) distribution of the electric field in a short-circuited waveguide (dashed lines identify the position of the specimen), $E_0$ = 17.4 kV/m; b) distribution of the heat field within the specimen, $T_{me}$ = 1800 K.

With an increase in the power flux density at the initial stage we also note a monotonic rise in the temperature of the central portion of the specimen (Fig. 2, curve 2). Numerical estimates and experimental data show that this stage may last for several hours, until the temperature experiences a sudden increase in temperature to the melting point $T_{me}$ of the material.

Let us examine in greater detail the conditions which lead to this sudden jump in temperature and to the localization of heating. We will change Eq. (2) to dimensionless form, using the following variables:

$$\Theta = \frac{T - T_0}{T_m - T_0}, \quad \dot{\varphi} = \frac{\dot{E}}{E_0}, \quad \eta = \frac{x}{l}, \quad \tau = \frac{t}{t_0}.$$

Here $t_0 = c\rho\ell(T_m - T_0)$ is the characteristic time of the process. Equation (2) assumes the form

$$\frac{\partial\Theta}{\partial\tau} = \frac{\partial}{\partial\eta}\left(\alpha\frac{\partial\Theta}{\partial\eta}\right) + \beta|\dot{\varphi}|^2, \tag{3}$$

where $\alpha = k(T_m - T_0)/S_0\ell$ and $\beta = 2\pi\ell\varepsilon''/\lambda_0$ are dimensionless parameters characterizing the thermophysical and electrophysical properties of the material.

The volumetric heating criterion $\gamma$, presented in the form of the ratio $\alpha/\beta$,

$$\gamma = A\frac{k(T_m - T_0)}{\varepsilon''} \tag{4}$$

enables us to establish the relationship between the quantity of heat transported by means of heat conduction and the thermal energy generated by the volumetric heat source. Here $A = \lambda_0/2\pi/^2S_0$ is a proportionality factor which links the characteristics of the electromagnetic field and the geometric dimensions of the specimen.

Using the numerical data in Fig. 1, we can trace the relationship between $\gamma$ and the temperature $T_m$. [The thermal conductivity of the material and the dielectric loss factor in expression (4) was determined for $T = T_m$.]

As we can see from Fig. 3, the curves $\gamma = f(T_m)$ attain their maximum at some critical temperature value $T_{cr}$. The numerical value of $T_{cr}$ is determined by the form of the temperature curves describing the thermal electrophysical properties of the given material.

The physical sense of the extremum nature of the curves $\gamma = f(T_m)$ lies in the fact that if the temperature of the material even locally exceeds the critical value (for the specimen under consideration this is 965 K), then even the slightest variation in temperature may lead to a cascade-like development of the process. This is associated with the fact that in the temperature region above the critical there occurs a temperature-dependent predominant increase in the quantity of energy generated by the volumetric heat sources, unlike the situation in the case of conductive heat transfer. Therefore, the formation of a stable equilibrium thermal distribution is possible when $T \le T_{cr}$.

The results of the numerical modeling make it possible to approach the solution of the important practical problem of thermal process stability from the standpoint of its effect on the dielectric of the SHF field. As follows from expression (4), there exists fully defined threshold relationships between the power of the SHF emissions and the thickness of the dielectric plate:

$$S_0 = \frac{\lambda_0}{l^2} \; \frac{k(T_{cr})(T_{cr} - T_0)}{2\pi\gamma_{cr}\varepsilon''(T_{cr})}, \tag{5}$$

where $\gamma_{cr}$ is the limit value of the volumetric heating criterion at which the relationship between the generated energy and the conductive heat transfer is such that there exist conditions for the establishment for thermal equilibrium within the dielectric. This quantity is a function of the material properties, the length of the electromagnetic wave, the given conditions of heat exchange, and it can be determined from the results of numerical modeling.

The curve for $S_0 = f(\ell)$ (Fig. 4) constructed with expression (5) makes it possible to isolate the region of such SHF power values and plate dimensions (the crosshatched portion) for which local material heating is unavoidable.

We will use the calculation experiment to trace the formation and development of a local thermal region within the dielectric specimen (Fig. 5). A standing electromagnetic wave with nonuniform three-dimensional distribution of the energy field acts on the ceramic-material plate contained within a short-circuited waveguide section.

In the initial stage of the process the thermal field within the material to some extent replicates the shape of the standing electromagnetic wave. The temperature is increased predominantly at the points corresponding to the maximum strength of the electric field. On passage through the critical point the rise in temperature assumes a cascade-like nature. The temperature variations in the electrophysical properties of the dielectric lead to a sharp increase in SHF energy absorption in the local, more highly heated zones.

The temperature in the region of elevated heat generation attains the melting point of the material within a relatively short interval of time ($\Delta t_{me} \ll t_0$).

The boundaries of the local melting region being formed are unstable. The melted segment of the dielectric screens out the electromagnetic emission, thus leading to a significant change in the shape of the electromagnetic field. The maximum of the electric component is displaced in the direction toward the radiation source. The intensive generation of heat in the zone of the maximum promotes the motion of the melting front, primarily in the direction of electromagnetic wave incidence.

Some of the motion in the opposite direction is associated with the high thermal conductivity and the superheating of the melt. As the melting front approaches the surface of the specimen, the thermal losses to the surrounding space increase and an ever-increasing portion of the thermal energy being developed is expended on the maintenance of the melt region already in existence. The motion of the melt front is gradually curtailed, and the thermal processes within the dielectric reach a state of stable equilibrium.

## NOTATION

$\dot{E}$, complex amplitude of the electric field; x, coordinate; $\omega$, angular frequency; $\lambda_0$, length of the electromagnetic wave in free space; $\varepsilon_0$, $\mu_0$, system coefficients; $\dot{\varepsilon}$, $\dot{\mu}$, complex dielectric permittivity and magnetic permeability of the medium; $\varepsilon'$, $\varepsilon''$, real and imaginary parts of the dielectric permittivity; $S_0$, power flux density of the incident electromagnetic wave; T, temperature; $T_0$, temperature of the ambient medium; $T_m$, maximum temperature of the steady state; t, time; $\Delta t_{me}$, time from the onset of localization to the melting of the material; c, ρ, k, the heat capacity, the density, and the thermal conductivity of the dielectric; $\alpha$, heat-transfer coefficient; $c_s$, the Stefan—Boltzmann constant; z, emissivity of the dielectric; Θ, φ, η, τ, dimensionless temperature, electric-field strength, a coordinate, and time.

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# AN ANALYTICAL MODEL OF THE STRESS—STRAIN STATE OF AN AXISYMMETRIC ELASTIC BODY UNDER CONDITIONS OF A TWO-DIMENSIONAL TEMPERATURE FIELD


A. M. Stolin, L. S. Stel'makh, and N. N. Zhilyaeva



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We have derived an analytical solution for the thermoelasticity problem involving the stress—strain state of an axisymmetric body subjected to the action of a two-dimensional temperature field.


We are called upon to deal with the problem of studying the stress—strain state (SSS) of a cylindrical elastic body in the presence of a temperature distribution that is a function of two spatial coordinates and of time, T = T(r, z, t) [1, 2]. This problem is particularly urgent for combustion-engineering processes which take place under markedly nonsteady and nonisothermal conditions [3]. Propagation of the combustion front over the specimens in these processes result in nonuniform thermal effects both in the lateral and longitudinal directions.

If the temperature is a function solely of one coordinate r and the time t, T = T(r, t), we generally make use of the analytical solutions for plane thermoelasticity problems [2, 4, 5]. Let us note that the ability of the models to solve this problem is based on the hypothesis of plane sections. Within the framework of this hypothesis, for an SSS symmetrical relative to the z axis it is possible to determine only the normal stresses, whereas the tangential stresses are assumed to be equal to zero. If the thermal effects are nonuniform along the length of the cylinder, the plane sections undergo bending. In this case, the tangential stresses may prove to be significant and they cannot be ignored. The problem becomes more complicated if we take into consideration the two-dimensionality of the temperature field and for the solution of the problem we generally make use of numerical methods. At the same time, for a number of questions which require both qualitative and quantitative investigation, it might prove to be useful to have an analytical solution of the three-dimensional problem. Among these questions we can point to the following: determining the criterial


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